Injection from Thermal Equilibrium and Acceleration to High Energy with Chaotic Gun Effect

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Abstract

Efficient acceleration mechanisms have been proposed in order to explain high energy spectra observed from astrophysical objects such supernova remnants, active galactic nuclei and X binaries. Most of them require an initial mechanism able to accelerate charged particles from thermal equilibrium to the onset of the stochastic process. The present paper argues the terms in which a numerical experiment, namely the chaotic gun effect, can be adopted as an injection mechanism. We estimate the injection energy and find it to be in the order of some MeV. We provide a proof which can support the power law dependence of spectrum of the charged particles' number on the energy in the chaotic region. Our arguments advocate that the validity of the main feature of an acceleration mechanism is displayed by chaotic gun effect: the spectrum.

Keywords: acceleration, synchrotron emission, gamma rays

Introduction

The fundamental assumption of the theory of diffusive shock acceleration is that accelerated particles diffuse in space - i.e. that the particle flux is proportional to the gradient of the particle density (Fick’s law). Charged particles deflected by fluctuations in the electromagnetic fields obey this relation only if their velocities are distributed almost isotropically. More precisely, the theory employs an expansion in the ratio of the plasma speed in the shock frame to the particle speed, and the velocity anisotropy is taken to be in the first order in this small parameter. At a shock front, the downstream plasma speed is of the same order as the thermal speed of the ions in the plasma. The question of how particles might be accelerated from the thermal pool up to an energy where they can be assumed to diffuse is referred to as the „injection problem”, and cannot be treated within the framework of the diffusive acceleration theory.

The injection-energy threshold for the Fermi process has to be larger than the mean kinetic energy of the electrons in the case of plasma identical with the electron thermal velocity. But in thermal equilibrium they practically do not accelerate. Thus, prior injection as a result of the action of some additional mechanism is always necessary for the statistical acceleration of the electrons.

Kundul and Vlachos proposed a scenario where the radiation of a small population of non thermal electrons can be reabsorbed from the same electrons (self absorption) or from the background (thermal) electrons through gyro-resonance absorption, and free-free absorption
Bosch-Ramon et al. advocated the first order Fermi acceleration mechanism as a source of the injection [4]. In order to explain the X-ray spectrum from Coma cluster Dogiel assumed a prior acceleration of the electrons, due to the Coulomb scattering in the thermal pool of electrons, up to a minimum energy of injection at which point Maxwell distribution switches into a power law distribution [6]. He found an alteration of the maxwellian spectrum due to the runaway flux of thermal particles into the region of acceleration which leads to an overestimate of the characteristic energy of the accelerated electrons.

Dieckmann et al. show that shock-reflected ion populations in the upstream plasma can drive collective instabilities, such that the resultant waves excited in the plasma damp on thermal electrons, thereby accelerating them across the magnetic field to mildly relativistic energies. A fully self-consistent treatment of this fundamentally nonlinear plasma process is obtained using large-scale particle-incell simulations, and there is close quantitative agreement with the analytical theory where points of contact exist. The ion population parameters are initialized on the basis of shock acceleration models and parameters, for example those of Cargill and Papadopoulos [13]. Thereafter, broadband electrostatic field oscillations grow in frequency range between the electron plasma- and gyro-frequencies, excited initially by Buneman-type instabilities, with episodes of high field temporally correlated with episodes of electron energisation. Several resonant and non-resonant mechanisms for the latter appear to be at work, of which the strongest involves stochastic wave-particle interactions [13].

If the relativistic electrons have an anisotropic distribution a maser effect will arise for high-frequency transverse radiation. Instability - the maser effect - will set in even if the relativistic electrons have a relatively small anisotropy. The increments for the development of the instability are of the same order as the damping decrements which prevail in the case of the perfectly isotropic relativistic - electron distribution [21]. The arguments used in favor of the dynamical dominance of magnetic fields over large spatial scales include the high linear polarization of kiloparsec-scale jets; the maser emission displays such linear polarization.

However turbulence in a magnetized plasma can lead to wandering or braiding of the magnetic field lines [7] which is a feature of a stochastic field. The turbulence could also be responsible for acceleration of the relativistic electrons producing nonthermal radiation in kiloparsec-scale jets [4].

Protheroe argued that, while the origin of the highest energy cosmic rays is still uncertain, it is not necessary to invoke exotic models such as emission by topological defects to explain the existing data [17]. Reynolds interpreted the non-thermal X-ray emission from SN 1006 as synchrotron emission by electrons accelerated in the remnant up to energies as high as 100 TeV [18], although Donea and Biermann suggested it may be brehmsstrahlung from much lower energy electrons [17]. Cosmic rays with energies up to 100 TeV are thought to arise predominantly through shock acceleration by supernova remnants (SNR) in our Galaxy [15]. In order to explain data for higher energies, i.e. to 1 EeV, some authors assumed reacceleration of the supernova component while still inside the remnant [3].

Argyris and Ciubotariu proposed a new and promising mechanism to accelerate low energy charged particles to high energies, namely chaotic gun effect [2]. In the next section we will reproduce some analytical features of the chaotic gun mechanism; a more comprehensive picture is readily found in the original paper. Then, in the third section, we argue how the numerical experiment of the chaotic gun effect can support the injection mechanism scenario. We estimate the injection energy and we compare it with the energy caused by the fluctuations of the field. Finally we outline some of the unsolved problems concerning our model and also outline that our model can support some observed data.
Acceleration Mechanism

A combination of cosmic electromagnetic fields (a cosmic source of energy), static magnetic fields, and cosmic charged particles (which have a general cosmic presence) can be used to accelerate charged particles to high energies in a limited space of a flight vehicle. The comptonization of ubiquitous cosmic (real) photons can be achieved by a second order Fermi (stochastic) acceleration mechanism which extracts energy from electromagnetic fields.

The region where the charged particles are accelerated, we will enforce to be finite and to concern a static magnetic field \( \vec{B}_0 \) along z axis:

\[
\vec{B} = (B_{0x}, B_{0y}, B_{0z}) = (0,0,B_0).
\]

A beam of charged particles gyrates in this magnetic field along a spiral trajectory. But a charged particle moving at an angle to an external magnetic field behaves as a non-linear oscillator which can generate an electromagnetic field.

We will assume that the beam posses a symmetry such that it generates a perturbation of an electromagnetic field with harmonic space-dependence and, generally, non-harmonic time-dependence of the form [2]:

\[
E_b(x,t) = (E_{bx}, E_{by}, E_{bz}) = (\text{Re}[E_x(t)e^{ikx}], \text{Re}[E_y(t)e^{iky}], 0) \quad \text{(2)}
\]

\[
B_b(x,t) = (B_{bx}, B_{by}, B_{bz}) = (0,0,\text{Re}[B_b(t)e^{ikx}]), \quad \text{(3)}
\]

where

\[
\vec{k} = (k_x, k_y, k_z) = (k,0,0) \quad \text{(4)}
\]

is the corresponding wave vector and \( \text{Re} \) applied to a quantity refers to the real part of that quantity. Using a test particle picture, Argyris and Ciubotariu showed that this field accelerates charged particles upon high energies.

The accelerating process is described by a set of coupled nonlinear equations for electromagnetic fields and charged particles, which are similar with the equations for three coupled oscillators; equations (44) and (45) from [2] display a first order Fermi mechanism of acceleration.

Nonlinear forces acting upon charged particles can accelerate it asymptotically [20]. An efficient mechanism of acceleration is obtained in the particular case when time dependence of the wave is (with dimensionless values):

\[
H_b(T) = \frac{1}{k_c} \frac{q B_s(t)}{mc} = \frac{1}{k_c} \frac{q E_{s,\gamma}(t)}{mc} = He^{-i\tau}, \quad H = \text{const.} \quad \text{(5)}
\]

where and the plasma frequency vanish and no electric field is on the x axis.

The acceleration takes place trough a “web structure” with a low limit of the wave amplitude A similar mechanism was suggested in an earlier paper by Melrose [16]. The charged particle accelerates when the wave frequency is an harmonic of the gyrofrequency of the charged particle in the rest of the frame of the charged particle’s gyration center. In his model, Melrose assumed that the wave is due to the microinstabilities occurring in the plasma. He concluded that a small fraction of the thermal particles can escape to the acceleration region depending on the thermal number density of the charged particles and also on their mass.

Numerical simulations showed that near resonances between cyclotron frequency harmonics and wave frequency a gun effect erupts, i.e. the charged particle is suddenly expelled to a
trajectory with larger radii. This is the case when \( H > 0.6 \). For smaller amplitudes the motion keeps the regular gyromagnetic behavior. Before the expulsion charged particle can extract energy from the magnetic field either as isolated resonance or overlapping resonances of magnetic field and gyration. The latter case involves \( H > 0.6 \). Numerical simulations exhibit chaotic modulation of the amplitude of particle momentum during the acceleration process due to the interactions between nonlinear oscillators.

The stochastic web may disappear because of the strong nonlinearity of the physical system. The instability appears as a conversion of an elliptical point into a hyperbolic one. Two new elliptical points of the doubled period are then formed and this represents an island-doubling bifurcation. If the energy of particles grows continuously there occurs a cascade of successive island-doubling bifurcations (a characteristic of Hamiltonian systems) in the vicinity of the elliptical points, which form necklaces of new (smaller) stability islands corresponding to higher-order resonances in the interaction of the particle with the wave. When the amplitude of the wave increases the stochastic net covers the whole phase plane and even the particles with small energies can diffuse into the region of very high energies. In other words, the time for the decoupling of correlations or the time for the loss of memory of the initial conditions is very small. In contrast with this situation, when one approaches the boundary of a stochasticity region, the diffusion is slowing down due to the finite correlation decoupling time [5].

### The Injection Energy

A beam of charged particles gyrates in this magnetic field along a spiral trajectory. But a charged particle moving at an angle to an external magnetic field behaves as a non-linear oscillator which can generate an electromagnetic field. In our model, we assume that the leptons in the jet dominate the radiative processes related to the gamma-ray production. The relativistic population of electrons flowing away into the jet, is exposed to the synchrotron photons emitted by the electrons, as we consider the magnetic field in our model. The energy losses of the relativistic leptonic plasma within the gamma-jet are mainly due to synchrotron emission.

In our model the charged particles undergo a complex motion in the electromagnetic field, which has a spatial harmonic dependence and a random dependence on time. To date many authors claim the nonlinear and also turbulent fields as sources of acceleration in astrophysical objects. Turbulence in a magnetized plasma can lead to wandering or braiding of the magnetic field lines [7].

Equations adopted by Argyris and Ciubotariu display a first order Fermi mechanism of acceleration [2].

Argyris and Ciubotariu found that even low energy electrons are accelerated to high energy when the dimensionless amplitude meets \( H > 0.9 \). In such case, i.e. low energy electrons, it raises the question how low the electron energy might be. In our model we will adopt the following scenario: the fluctuations of the electromagnetic field at thermal equilibrium assign an amount of extramagnetic field to the electrons, this amount is absorbed by electrons, which move in a regular regime i.e. they meet the onset of a stochastization and thereafter they accelerate according to the chaotic gun effect mechanism. The radiation emitted from a small population of non-thermal electrons can be reabsorbed from the same electrons (self-absorption) through gyro-resonance absorption. In an early paper, Kundu and Vlachos suggested that the non-thermal electrons can be unstable and these instabilities can be the source of very high brightness temperature, fine structure pulsations [14].

Therefore instead of the assumption made by Enßlin et al. [6] we will assume that the regular chaotic gun effect provides the mechanism necessary to supply the jump from the thermal
regime to injection energy since the chaotic gun effect is a self consistent process of acceleration. In the following we will estimate the range of the injection energy of the electron. According to [2] the essential parameter in obtaining a gun effect is the gyrofrequency ratio:

$$\beta = \frac{H}{\Omega_B} = \frac{[\Omega_B]}{\Omega_B},$$

(6)

where $$\Omega_B$$ is the nonrelativistic dimensionless gyrofrequency in the static magnetic field of strength $$B_0$$:

$$\Omega_B = \frac{1}{\kappa c} \frac{qB_0}{mc},$$

(7)

and $$\Omega_b$$ is the dimensionless gyrofrequency of the magnetic field generated by the electromagnetic wave:

$$\Omega_b = \frac{\omega}{kc}.$$  

(8)

Let us label $$\beta_{inj}$$ the $$\beta$$ parameter value at which the onset of the chaotic motion occurs and hence an efficient acceleration starts. For the $$n$$th order of the resonance the charged particles' energy is:

$$\varepsilon_n = nmc^2 \frac{H}{\beta},$$

(9)

where the order of the resonance expresses how many times the frequency of the emitted radiation is larger than the relativistic cyclotron frequency is:

$$n = \frac{\omega}{\omega_c} = \frac{\gamma \omega}{\omega_B}.$$  

(10)

In the above equation, $$\omega_B = \frac{eB_0}{mc}$$ is the nonrelativistic cyclotron frequency. Therefore, using (9) and (10) one can express the injection energy as:

$$\varepsilon_{inj} = mc^2 \frac{H_{inj}}{\beta_{inj}} \frac{\omega}{\omega_c} \approx 0.5\text{MeV} \frac{H_{inj}}{\beta_{inj}} \frac{\omega}{\omega_c}.$$  

(11)

Argyris and Ciubotariu found $$\beta_{inj} = 4$$ for $$H_{inj} = 0.2$$. The maximum in the spectrum of radiation from individual electron occurs at the frequency [11]:

$$\omega \approx 0.07 \gamma^3,$$  

(12)

which is expressed in CGS. In SI the coefficient 0.07 must be replaced by 35.

Substituting (12) in (11) one finds $$\varepsilon_{inj} \approx 0.378\text{ MeV}$$, which corresponds to $$\gamma \approx 0.756$$. The kinetic energy of the electron is $$\varepsilon_{kin} \approx 0.021 \times 3kT$$.

In an attempt to ascribe the injection to the bremsstrahlung loss of the suprathermal electrons accelerated by turbulence within the medium, Sarazin and Kempner assumed the nonthermal population to consist of electrons with kinetic energies $$\varepsilon_{kin} > 3kT$$ and the total number of
nonthermal particles to be 1% of the thermal population of electrons [19]. The data observed by
ROSAT found the central gas averaged temperature in Coma cluster to be of the order \( kT \sim 8\text{keV} \). Assuming the Coulomb collisions to be responsible for thermal equilibrium, Dogiel
found the injection energy \( \equiv 0.9\text{MeV} \) [6].

An integro-differential equation of the motion of a charged particle is [12]:

\[
m \ddot{V}(t) = \int_{0}^{\infty} e^{-\frac{s}{1\tau_{r}}} \mathcal{F}(t + \tau) ds,
\]

where \( s = \frac{1}{\tau_{r}}(t' - t) \) is a variable which contains the interval \( t' - t \) on which one integrates and this
interval is of the order \( \tau \). If the force would act in a small interval of time, e.g. \( \sim 2\pi \times 10^{-24}\text{s} \),
then there is an energy uncertainty due to the imprecision in determining the energy. The
uncertainty is of the order:

\[
\Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.25 \times 10^{-34} \text{J} \equiv 10^{-10} \text{J}}{2\pi \times 10^{-24} \text{G}} \equiv \frac{10^{-10}}{1.6 \times 10^{-19}} \text{eV} \equiv 10^{6}\text{eV}.
\]

If this amount of energy would correspond to an extramagnetic field then the strength of the
extramagnetic field would be:

\[
\Delta B = 10^{-1} \sqrt{10^{-1} (8\pi)^{1/2} G} \equiv 3 \times 10^{-3} \text{G}.
\]

This amount of extramagnetic field could be the magnetic field absorbed by the electron due to
the fluctuations of the field if this is the case. This value is \( 10^{3} \) times larger than the usual
assumed strength of the magnetic interstellar medium \( 10^{-6} \text{G} \).

From (6) labeling \( \beta_{inj} \) the critical value of the parameter \( \beta \) when the onset of the chaotic
motion of the charged particle occurs one can estimate the strength \( B_{inj} \) of the magnetic field
of the wave produced by the beam:

\[
B_{inj} = \beta_{inj} B_{0}.
\]

Numerical experiments showed that \( \beta_{inj} \) is of the order 4. That is \( B_{inj} \) is 4 times higher
than \( B_{0} \).

Once the chaotic motion starts, a fraction of the charged particles of the beam accelerates along
the wave vector direction. Their acceleration is supplied by the self absorption mechanism.
During chaotic acceleration, the charged particles emit synchrotron radiation. The coherence
length of the emission can be estimated by:

\[
l_{\text{coher}} \equiv k_{m}^{-1},
\]

where \( k_{m}^{-1} = \gamma \omega_{h}^{-1} \). Assuming \( B_{0} = 10\mu\text{G} \) and \( \gamma \equiv 16.9 \) we find \( l_{\text{coher}} \equiv 0.09 \cdot 10^{9}\text{cm} \).

The Spectrum of the Synchrotron Emission

Assuming a special kinetic equation in an earlier paper [8] we found a power law shape of the
spectrum emitted by charged particles accelerated by chaotic gun effect, i.e., where is the
frequency and is the spectral index. Numerical simulations performed for the spectrum lead us
to a good approximation of the differential index for the high and the low flux spectrum,
respectively, in the energy region from 1 TeV to 5 TeV, which could match the time-averaged energy spectrum of Mrk 501 [1].

**Discussion and Conclusions**

Our derivation of the kinetic equation, and then of the spectrum, may seem to be redundant, since the chaotic gun effect is a self-consistent process and also the spectrum obtained must be similar to that assumed in the case of the magnetic inhomogeneities. But the derivation provides us with a proof that above the injection the random electromagnetic field is due to the synchrotron emission. It also provides a conditional probability for the electron, which in fact is a cross section for this special Compton scattering during chaotic motion of the electron.

We did not take into account in our model the magnetic reconnection. This process may play an important role in producing gamma rays.

Chaotic gun effect has a hyperbolic behavior. In our derivation of the correlation tensor of the second order which describes the conditional probability of the electron state, we assumed a cutoff of the maximum energy, but we did not establish it. On the other hand, we stated that the averaged electric field between two states of the electron is responsible for acceleration. A non-static electric field can directly accelerate a charged particle and such field can be established by a time-varying magnetic field

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \]  

(18)

For a characteristic length scale over which the field varies, one can use the following order of magnitude calculation:

\[ \frac{E}{L} \sim \frac{Bc}{L} \Rightarrow E \sim Bc. \]  

(19)

The total energy that can be given to a particle is:

\[ \gamma mc^2 \sim \int qEdl \sim qBcL, \]  

(20)

where \( L = l_{\text{coher}} \). Adopting, for instance, \( l_{\text{coher}} \equiv 0.09 \cdot 10^9 \text{cm}, \) \( B_0 = 10^{-6} \text{G} \) one obtains \( \gamma mc^2 \sim 0.27 \cdot 10^{13} \text{eV} \).

Due to its self consistency, the chaotic gun effect is very flexible to the choice of the turbulence type of the electromagnetic field into which charged particles move. But the flexibility concerns also the absence of a physical reason to adopt a certain type of turbulence.

The observed data concerning the spectral index variation of the spectrum of gamma-rays could be explained by the stability of each kind of turbulence and also by the variation of the averaged magnetic field in the source.

The chaotic gun effect can supply the diffusive shock acceleration mechanism as an injection mechanism, but it also could be assumed the only mechanism for certain sources or astrophysical objects, e.g. AGN jets since it displays magnetic inhomogeneities of large scales.

The chaotic gun effect can provide a conical shape to the jet geometry for regular regime [9]; more than this, it can also support the observed data which display that the source of the X rays is a extended halo of the radio source since during acceleration process the radii of the gyration increases [6].
Fleishman and Melnikov showed that for low values of the parameter $Y = \frac{\omega_p}{\omega_B}$, smaller than 0.3, the coherent electron cyclotron maser emission takes place [10]. This is in accordance with one of the chaotic gun effect assumptions, namely the vanishing of the electron plasma frequency. On the other hand, this assumption is in accordance with the theoretical model of the synchrotron radiation for relativistic electrons [11].

References

5. Ciubotariu C. et al. - Extracting Energy from Cosmic Electromagnetic Fields and Plasmas by a Chaotic Gun Effect for Relativistic Charged Particles, personal communication
Injecție și accelerare de la echilibru termic la energii înalte cu efect tun haotic

Rezumat

De-a lungul timpului au fost propuse mecanisme eficiente de accelerare cu scopul de a explica spectrele de energii înalte provenite de la obiecte astrofizice cum ar fi rămâșițele de supernova, nucleele active galactice și binarele X. Aproape toate modelele ar fi necesitat un mecanism inițial care să accelerze particulele încărcate de la echilibru termic până la declanșarea procesului stocastic. În lucrarea de față sunt prezentate argumentele pentru care un experiment numeric, numit efect tun haotic, ar putea fi adoptat ca un mecanism de injecție. În lucrare se estimează energia de injecție și se găsește a fi de ordinul câtorva MeV. Propunerea este susținută și de faptul că spectrul emisiei particulelor încărcate este de tip putere, care este în general principalul argument care poate valida mecanismul.